

Objectives

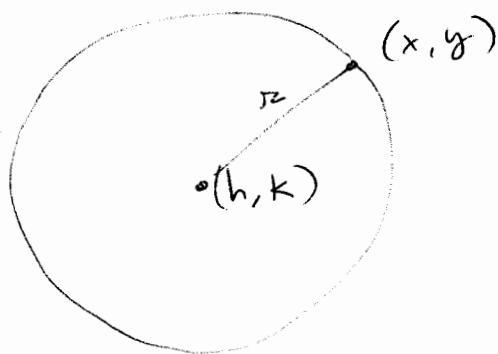
1) Graph circles from equations in standard form $(x-h)^2 + (y-k)^2 = r^2$

2) Write equations of circles in standard form when given as

$$ax^2 + bx + cy^2 + dy = e$$

3) Write equations of circles in standard form given center (h, k) and radius r .

A circle is a set of all points (x, y) that are the same distance (called the radius) from a single point (called the center (h, k))



distance formula

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

square both sides

$$r^2 = (x-h)^2 + (y-k)^2$$

change sides

$$(x-h)^2 + (y-k)^2 = r^2$$

This is called the standard form of the equation of a circle because we can determine the center and radius by looking at the equation.

Graph.

Yes

① $(x+3)^2 + (y-1)^2 = 25$

$$\uparrow$$

 $(x-h)=0$

$$x=h$$

$$\uparrow$$

 $x+3=0$

$$x=-3$$

X coord
of center

$$\uparrow$$

 $y-k=0$

$$y=k$$

$$\uparrow$$

 $y-1=0$

$$y=1$$

Y coord
of center

$$r^2$$

$$\sqrt{r^2} = r$$

$$\uparrow$$

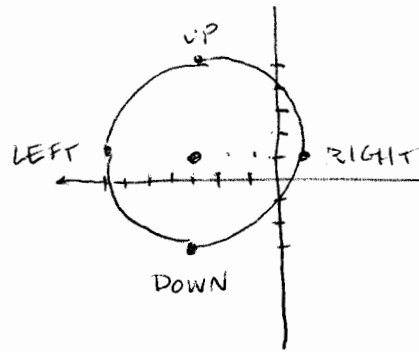
 $\sqrt{25} = r$

$$r=5$$

step 1: Find
coordinates
of center
and value
of radius.

① cont graph.

step 2: plot center
 $(-3, 1)$



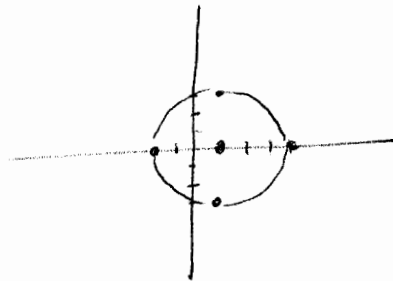
step 3: use radius
 to plot four
 points left, right, up, and down from
 center.

step 4: Connect the four dots with smooth round
 curves to make a circle.

YES

② $(x-1)^2 + y^2 = 9$

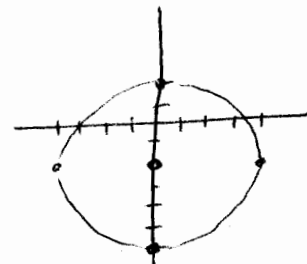
$x=1$ $y=0$ $r=\sqrt{9}=3$
 $(1, 0)$



SKIP

③ $x^2 + (y+2)^2 = 16$

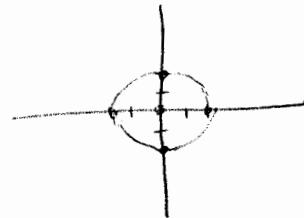
$x=0$ $y=-2$ $r=\sqrt{16}=4$



SKIP

④ $x^2 + y^2 = 4$

$x=0$ $y=0$ $r=\sqrt{4}=2$



Note: Circles can be graphed on GC if you solve
 the equation for y :

$$y^2 = 4 - x^2$$

$$y = \pm \sqrt{4 - x^2} \quad \leftarrow 2 \text{ functions}$$

$$y_1 = \sqrt{4 - x^2}$$

$$y_2 = -\sqrt{4 - x^2}$$

ZOOM SQUARE (5) makes it rounder

So it's not difficult to graph a circle if you know the center and radius. But how do we find them if we don't?

Write each equation in standard form and find the center and radius.

YES (5) $x^2 + y^2 + 4x - 8y = 16$

step 1: Collect x^2 and x together
 y^2 and y together
 and constants on RHS.

$$x^2 + 4x + y^2 - 8y = 16$$

step 2: Complete the square in x AND
 complete the square in y .

$$\# = \frac{4}{2} = 2$$

$$\#^2 = 2^2 = 4$$

for x

$$\# = \frac{-8}{2} = -4$$

$$\#^2 = (-4)^2 = 16$$

for y

* Be sure to add both CTS $\#^2$ to the RHS *

$$x^2 + 4x + 4 + y^2 - 8y + 16 = 16 + 4 + 16$$

$$(x+2)^2 + (y-4)^2 = 36$$

center $(-2, 4)$

radius 6

YES (6) $2x^2 + 2y^2 = \frac{1}{2}$

divide all terms by 2:

$$x^2 + y^2 = \frac{1}{4}$$

$$(x-0)^2 + (y-0)^2 = \left(\frac{1}{2}\right)^2$$

center $(0, 0)$

radius $\frac{1}{2}$

Given the center and radius, write equation of the circle in standard form.

SKIP ⑦ center $(-7, 3)$ radius 10.

step 1: write the standard form as a formula

$$(x-h)^2 + (y-k)^2 = r^2$$

step 2: substitute and simplify r^2

$(x-h)^2$ and $(y-k)^2$ can remain unsimplified.

$$(x - (-7))^2 + (y - 3)^2 = 10^2$$

$$(x+7)^2 + (y-3)^2 = 100$$

combined version

YES ⑦+8 center $(-7, 3)$ and radius $\frac{2}{3}$

$$(x+7)^2 + (y-3)^2 + \left(\frac{2}{3}\right)^2$$

$$(x+7)^2 + (y-3)^2 = \frac{4}{9}$$

or

$$9(x+7)^2 + 9(y-3)^2 = 4$$

SKIP ⑧ center $(0, -2)$ radius $\frac{2}{3}$

$$(x-0)^2 + (y+2)^2 = \left(\frac{2}{3}\right)^2$$

$$x^2 + (y+2)^2 = \frac{4}{9}$$

might also clear fractions:

$$9x^2 + 9(y+2)^2 = 4$$

Extras: Write in standard form and graph.

SKIP ⑨ $x^2 + y^2 + 6x - 2y = 6$

$$x^2 + 6x + 9 + y^2 - 2y + 1 = 6 + 9 + 1$$

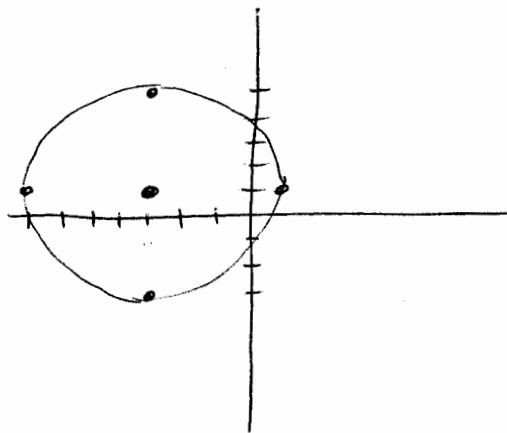
$$\left(\frac{6}{2}\right)^2 = 3^2 = 9 \quad \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$$

$$(x+3)^2 + (y-1)^2 = 16$$

center $(-3, 1)$

$$\text{radius } \sqrt{16} = 4$$

same graph as ①



Math 70 MG 5/e 10.1

SKIP

10

$$x^2 + y^2 + 6x + 10y - 2 = 0$$

$$x^2 + 6x + 9 + y^2 + 10y + 25 = 2 + 9 + 25$$

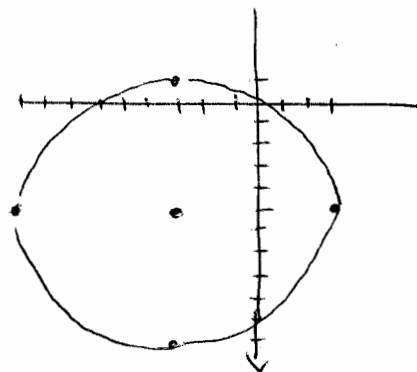
$$\left(\frac{6}{2}\right)^2 = 3^2 = 9$$

$$\left(\frac{10}{2}\right)^2 = 5^2 = 25$$

$$(x+3)^2 + (y+5)^2 = 36$$

center $(-3, -5)$

radius $\sqrt{36} = 6$



SKIP

11

$$x^2 + y^2 - 8y + 5 = 0$$

$$x^2$$

↑

$$\left(\frac{0}{2}\right)^2 = 0$$

$$+ y^2 - 8y + 16 = -5 + 16$$

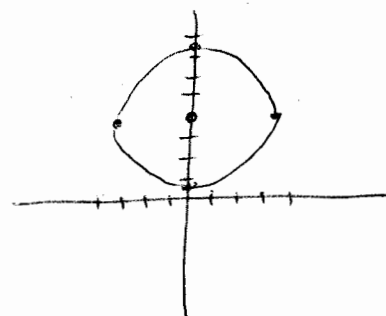
↑

$$\left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$$

$$(x-0)^2 + (y-4)^2 = 11$$

center $(0, 4)$

radius $\sqrt{11} \approx 3.3$



you won't be asked
to graph $r = \sqrt{11}$!

Graph.

(12)
SKIP

$$x = -2y^2 - 4y$$

Notice: No x^2 ! This is not a circle.

It's a parabola going left or right.

$$x = -2(y^2 + 2y + 1) - (-2)(1)$$

$$\# = \left(\frac{-2}{2}\right)^2 = 1^2 = 1$$

$$x = -2(y+1)^2 + 2$$

vertex $(2, -1)$
opens left
narrow

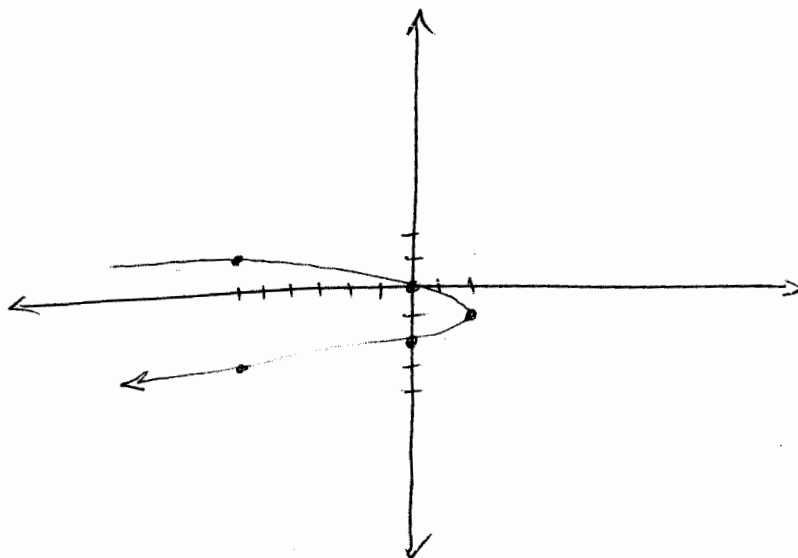
-OR-

vertex formula

$$y = \frac{-b}{2a} = \frac{-(-4)}{2(-2)} = \frac{4}{-4} = -1$$

$$x = -2(-1)^2 - 4(-1) \\ = -2 + 4 = 2$$

vertex $(2, -1)$
 $a = -2$ opens left
narrow.



Graph.

(13)
YES

$$x = y^2 + 6y + 2$$

no x^2 ! Again a parabola x & $y^2 \Rightarrow$ left or right

$$x = (y^2 + 6y + 9) + 2 - 9$$

$$\left(\frac{6}{2}\right)^2 = 3^2 = 9$$

$$x = (y+3)^2 - 7$$

vertex $(-7, -3)$ $a=1$

opens right

- OR -

vertex formula

$$y = -\frac{b}{2a} = -\frac{6}{2(1)} = -3$$

$$x = (-3)^2 + 6(-3) + 2$$

$$= 9 - 18 + 2$$

$$= -7$$

vertex $(-7, -3)$ $a=1$ opens right(14) Find the y -intercepts in (13).set $x=0$ solve for y .

$$0 = (y+3)^2 - 7$$

$$7 = (y+3)^2$$

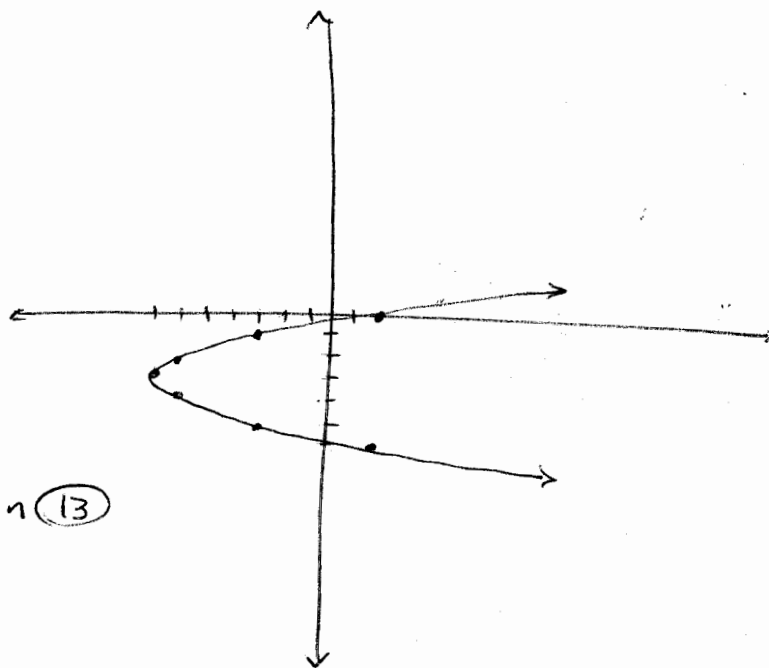
$$\pm\sqrt{7} = y+3$$

$$\boxed{-3 \pm \sqrt{7} = y}$$

(15) Find the x -intercept in (13)Set $y=0$ solve for x .

$$x = 0^2 + 6(0) + 2$$

$$\boxed{x=2}$$

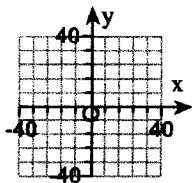
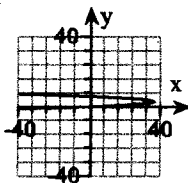
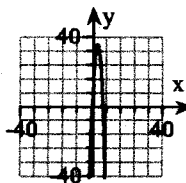
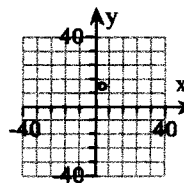


10.1.59

Sketch the graph of the equation. If the graph is a parabola, find its vertex. If the graph is a circle, find its center and radius.

$$x = -4y^2 + 24y$$

Choose the correct graph below.

☐ A.

☒ B.

☐ C.

☐ D.


Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

☒ A.

The graph is a parabola with the vertex located at .

(Type an ordered pair.)

☐ B.

The graph is a circle with radius .

The center is located at .

(Type an ordered pair.)

CTS

$$x = -4(y^2 - 6y + 9) + 36$$

-36

$$x = -4(y-3)^2 + 36$$

vertex (36, 3)

left because x, y^2 and $a = -4 < 0$.

Name _____

Date _____

TI-84+ GC 35 ZOOM Square for Circles

- Objectives: Graph several semi-circles simultaneously on the GC
 Use the VARS menu to refer to another y in the $Y=$ menu and save typing
 Use ZOOM Square to make the circles round on the GC screen
 Notice the limitation of GC graphs for circles

The VARS button has two menus: VARS and Y-VARS. Y-VARS refers the $Y=$ menu. If we have defined y_1 in the $Y=$ menu, and y_2 is the opposite of y_1 , we can use the Y-VARS menu to make $y_2 = -y_1$

- 1) What shape is the graph of $x^2 + y^2 = 25$?
- 2) Solve $x^2 + y^2 = 25$ for y , put these two equations into GC as y_1 and y_2 .

Y= **CLEAR** **2nd** **x^2** **2** **5** **-** **X,T,θ,n** **x^2** **)**

Define y_2 as $y_2 = -y_1$

(-) **Open VARS:** **VAR** **Move to Y-VARS:** **)** **Select Function:** **ENTER** **Select y_1 :** **ENTER** **Move to next:** **ENTER**

- 3) What shape is the graph of $y = -\sqrt{6.25 - x^2}$?

- 4) Put $y = -\sqrt{6.25 - x^2}$ as y_3 in GC.

(-) **2nd** **x^2** **6** **.** **2** **5** **-** **X,T,θ,n** **x^2** **)** **ENTER**

- 5) What shape is the graph of $(x+2)^2 + (y-2)^2 = 1$?

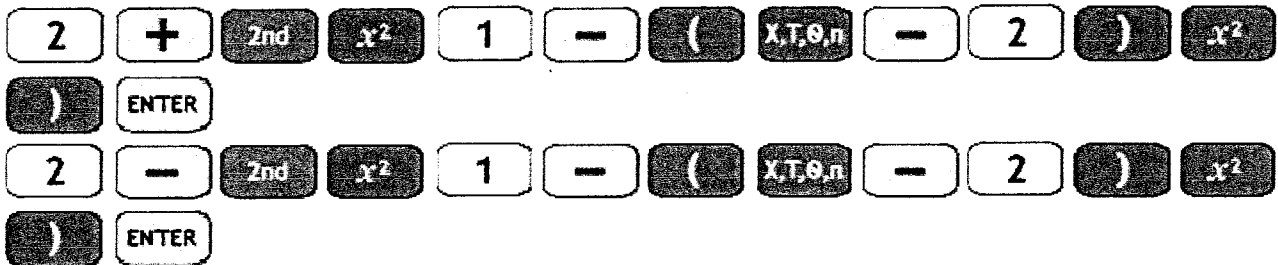
- 6) Solve $(x+2)^2 + (y-2)^2 = 1$ for y ; put these two equations into GC as y_4 and y_5

2 **+** **2nd** **x^2** **1** **-** **(** **X,T,θ,n** **+** **2** **)** **x^2**
) **ENTER** **2** **-** **2nd** **x^2** **1** **-** **(** **X,T,θ,n** **+** **2** **)** **x^2**
) **x^2** **)** **ENTER**

TI-84+ GC 35 ZOOM Square for Circles page 2

7) What shape is the graph of $(x-2)^2 + (y-2)^2 = 1$?

8) Solve $(x-2)^2 + (y-2)^2 = 1$ for y ; put these two equations into GC as y_6 and y_7



9) Graph in a standard window: **ZOOM** **6**. What errors does the GC make?

10) Graph in a square window: **ZOOM** **5**. Which error(s), if any, does this window fix?

11) Sketch the graph of all of these semicircles on the same axes. Use what you know about how the center and radius of a circle appear in its equation. Sketch the axes very lightly. You may wish to use a GC table from $x = -5$ to $x = 5$. Be sure your graph corrects the GC errors.

